Evaluating nonhydrostatic dynamics for global and regional ocean modeling

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1 The Challenge

Current state-of-the-art global ocean models, such as LANL's POP (Parallel Ocean Program), have been ideally suited to answering climate questions over centuries long timescales and under coarse resolution. With the aid of increased computational capabilities, these "primitive equation" models are increasingly being applied to problems of shorter, more pressing timescales and smaller, even regional, spatial scales. Increased spatial resolution and the corresponding newly resolved physical features, however, represent a "gray area" over which a key modeling assumption in the derivation of the primitive equations, i.e. the hydrostatic approximation, breaks down. This break down calls into question the validity of the primitive equations for use in the next generation of global ocean models and makes highly suspect the results and predictions of current primitive equation regional models.

My interest is in examining nonhydrostatic effects in high-resolution regional ocean studies which include variable ocean bottom (topography) and density profiles. The primary challenge of this research is to determine the range of applicability of hydrostatic ocean models for climate and regional applications, and estimate whether nonhydrostatic effects will be important enough to change model development from the hydrostatic primitive equations to the full nonhydrostatic equations.

This highlight attempts the following: 1) to frame the problem mathematically and 2) present a preliminary result.

2 The $3D$ Inviscid Boussinesq Equations

Initial analysis begins with the inviscid primitive equations **plus** the "nonhydrostatic" terms (underlined in below equations), as well as, linearization against a simple base state: $U(z, y)$ the zonal velocity, $\Phi(y, z)$ the pressure field normalized by a constant reference density (ρ_r), and $\rho(y, z)$ the density field. With small perturbations denoted by $'$,

$$u'_t + \mathbf{u_p}' \cdot \nabla \underline{M} = 0 \tag{1}$$

$$B_t + \mathbf{u_p}' \cdot \nabla \underline{Q} = 0 \tag{2}$$

$$v'_t + fu' + \Phi'_y = 0 \tag{3}$$

$$\underline{w'_t} - \underline{Fu'} + \Phi'_z - B = 0 \tag{4}$$

$$v'_y + w'_z = 0 \tag{5}$$

where the bouyancy $B \equiv -\rho' g/\rho_r$, $\mathbf{u_p} \equiv (0, v, w)$, $M \equiv U - fy + \underline{Fz}$ (absolute momentum), and $Q = -\underline{FU} + \partial\Phi/\partial z = -g\rho/\rho_r$ ($\propto$ potential temperature). Three results follow immediately: 1) the generation of u' depends on the angle of trajectories compared with base state M surfaces of slope M_y/M_z which contain a contribution from the nonhydrostatic coriolis term F, 2) the generation of B depends on the angle of trajectories compared with Q surfaces of slope Q_y/Q_z which also contain an F contribution, and 3) the zy-trajectories may be written in terms of a streamfunction, i.e. $w' = \psi_y$ and $v' = -\psi_z$.

In the manner of Hoskins (1978), several key and illuminating frequencies of the basic flow may be defined based on the Brunt-vaisala frequency N, the thermal wind relation S, and the base vorticity:

$$\tilde{N}^2 = N^2 - \underline{FU_z}, \quad S^2 = fU_z + \underline{FU_y}, \quad W_k^2 = f(f - U_y), \quad W_j^2 = f(\underline{F} + U_z). \tag{6}$$

Thus $M_y/M_z = -W_k^2/W_j^2$ and $Q_y/Q_z = -S^2/\tilde{N}^2$.

3 The Symmetric Instability

In the 1970s Stone showed that an initially steady sheared flow was unstable to three kinds of instabilities (each with a characteristic length scale), which dominate in different regions of Richardson number: when $R_i > .95$, the common baroclinic instability dominates, for $1/4 < R_i < .9$ the symmetric instability, and for $R_i < 1/4$, the Kelvin-Helmholtz instability. The symmetric instability has the appropriate length scale and (potentially) nonhydrostatic character to be of interest.

In a normal mode analysis of eqns 1–5, $\psi \propto e^{i\sigma t + kz}\Psi(y)$. Solving for the frequency σ, we find a necessary condition for symmetric instability:

$$\left(1 - \frac{U_y}{fo}\right) R_i < \left(\frac{DF}{U} + 1\right)^2 \tag{7}$$

Where $R_i = D^2(F^2 + N^2)/U^2$ for vertical spatial scale D plotted below.

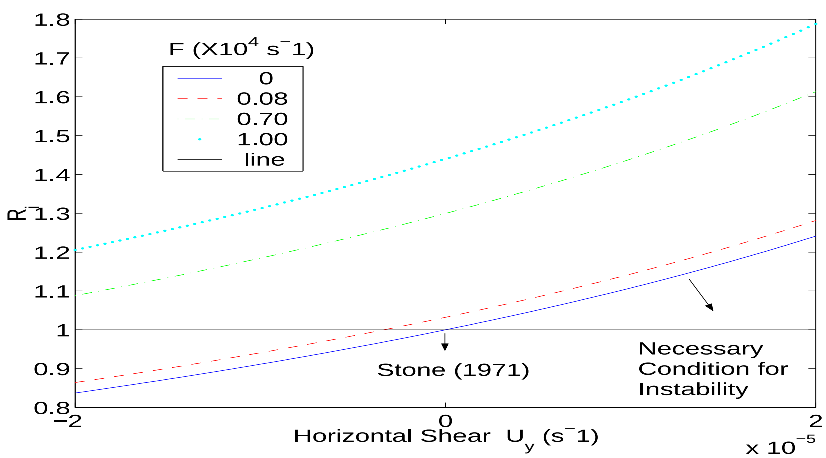

Figure 1: The nonhydrostatic Coriolis term F relaxes the R_i condition potentially decreasing the base flow stability.